PERGAMON

International Journal of Solids and Structures 39 (2002) 559–569

INTERNATIONAL JOURNAL OF SOLIDS and STRUCTURES

www.elsevier.com/locate/ijsolstr

Lamb mode stresses as means of identifying voids in the adhesive zone of bilaminates

R.Y. Vasudeva *, G. Sudheer

Department of Applied Mathematics, Andhra University, Visakhapatnam 530 003, India

Received 6 July 2001

Abstract

Lamb wave technique has emerged as a reliable tool in the nondestructive testing of laminated plates. Some current studies to identify the specific Lamb modes that can characterize different kinds of defects in layered plates using Lamb waves have shown that the modes for which high stresses and low displacements occur in the interface indicate the presence of defects like pores or voids whereas the modes for which the displacements are high show the presence of harder inclusions. In this context this paper tests an earlier analytical model developed to facilitate NDT of porosity in the adhesive zone of bilaminates.

The model tested treats the pore infested thin adhesive region as a linear elastic material with voids (LEMV). For certain parametric values of the LEMV adhesive layer the influence of these voids on dispersion and stresses carried by the first few Lamb modes in glass/glue/glass (G/g/G) bilaminate is traced in the range 0–10 MHz. The frequency–phase velocity points experimentally obtained by Kundu and Maslov are seen to fall very close to the present dispersion. The stresses traced using the present model in G/g/G plate at these experimentally tallied points show an easily discernable rise in the central region of adhesive, as observed by Kundu and Maslov.

The model appears to be useful as a good first approximation to detect voids in adhesive zone of composite structural elements.

Keywords: Lamb modes; Stresses; Porosity; Adhesive zone; Modeling

1. Introduction

> "...Guided waves in plates and rods which were previously mainly of academic interest now play an important role in such diverse fields as nondestructive evaluation and fiber optic devices..."
>
> B.A. Auld

Even a cursory look at present day literature on wave motion in elastic solids is sufficient to get a feel of the above remarks (Auld, 1986). The new dimensions the study of elastic wave propagation acquired in its

* Corresponding author. Fax: +91-891-544-305.
E-mail address: vasudevary@hotmail.com (R.Y. Vasudeva).

0020-7683/02/$ - see front matter
PII: S0020-7683(01)00159-7

applicability in recent years are truly fascinating. Signal processing, acoustic emission, seismology and ultrasonics are only a sample to mention.

The present paper is concerned with application of elastic wave motion in plates to nondestructive testing or evaluation of adhesive bond quality in composite bilaminates employing what is well known as the Lamb wave technique (Viktorov, 1967).

Lamb waves are waves that propagate in a freely vibrating plate. The unique properties of Lamb waves have made them increasingly attractive for nondestructive testing of bonded structures. It is now accepted that a large area of the structural element can be inspected using Lamb waves. The sensitivity and efficiency of adhesive bond inspection using Lamb waves has been the subject of study in recent years in many laboratories concerned with bond quality inspection. Bar-Cohen, Chimenti, Rokhlin, Mal, Pilarski, Rose and Achenbach among others made lasting contributions that are widely referred to in this area. A bibliography of papers covering various theoretical and practical aspects of Lamb waves is available on internet at www.NDT.net.

Modern technology employs tailor made composite materials to suit specific performance needs and very often adhesives are employed in fabricating them. Even in the instance of employing riveting as the major fastening tool in aerospace technology, we learn that adhesives are also used selectively to arrest propagation of cracks (Dattaguru, 1999).

One of the familiar reasons for bond failure in adhesively joined surfaces is the distributed porosity in the region of adhesion. If the extent of porosity exceeds an accepted level, the component has to be rejected. In any technique employed to nondestructively identify or estimate the extent of porosity a good mathematical model of the pore infested adhesive zone would be of immense help to the practicing NDT engineers. One of the various NDT tools for inspecting interface imperfections is the Lamb wave/leaky Lamb technique. The present paper is a continuation of the authors' attempt at developing a reliable analytical model of the interface — whose imperfection is due to a cluster of minute voids in the region of the glue — to aid LW/LLW technique. (We use pores and voids synonymously throughout the paper.) A void, in the context of adhesive bonding is, "any area that should contain, but does not contain adhesive" (Hegemier, 1989).

Coming back to Lamb waves and their utility in NDT of bond quality, these waves have infinitely many modes. In a given frequency range, however, there are a finite number of them. As they propagate in the plate medium, their phase velocity is affected by the frequency. Till recently this dispersion of Lamb modes that is obtained theoretically assuming some or other model for imperfect bond is compared with the dispersion obtained from experiments. But in recent years the attention is shifted to identifying those modes that can characterize specific defects like inclusions, harder or softer than the bulk medium (Yang and Kundu, 1998; Kundu and Maslov, 1997). One of the observations is that those modes that carry higher stresses can be utilized to identify softer or porous defects while those that carry higher displacements but lower stresses are used to detect harder inclusions (Kundu and Maslov, 1997). In particular the influence of interfacial defects on Lamb mode stresses in glass bilaminates was studied by Kundu and Maslov. The stress patterns for the symmetric and anti-symmetric modes are illustrated and discussed by these authors.

In order to develop a model of the adhesive region with distributed porosity Vasudeva and Govinda Rao (1991, 1992) and Vasudeva and Sudheer (in press) have employed the theory of linear elastic materials with voids (LEMV) (Cowin and Nunziato, 1983). Vasudeva and Govinda Rao (1991) used the LEMV model to describe the adhesive zone to identify porosity in the region of adhesion using Lamb modes. They obtained Lamb wave dispersion in A1/adhesive/A1 plates that compares closely with the one obtained employing a general imperfect bond model (Mal et al., 1989). They (Vasudeva and Govinda Rao, 1992) further extended this work to fiber-reinforced composite bilaminates in which the dispersion spectrum obtained is compared with the one in perfectly bonded plates. Later Vasudeva and Sudheer (in press) have shown that in the LLW technique Lamb wave attenuation is a useful parameter in identifying voids in adhesive interface zones. They inferred this by looking at how the attenuated leaky Lamb mode dispersion differs from that in a perfectly bonded plate (Dayal and Kinra, 1989) if vacuous imperfections were present. Following Vasudeva and Govinda Rao, Paul and Nelson (1996) analyzed the influence of voids in the interface on flexural vibrations of a bilayered composite hollow piezoelectric cylinder employing the LEMV theory.

Motivated by the findings of Kundu and Maslov (1997) the present work undertakes evaluation of stresses carried by the first few symmetric and anti-symmetric Lamb modes in a composite glass plate in order to appreciate the utility of the LEMV model in detecting porosity in the adhesive interface of the bilaminate. However what we present is not an exhaustive study nor is it quantitative in nature. We would like to note that the theory of LEMV does not consider either the pore size or how the pores are actually distributed in the medium. However, it attempts to incorporate the presence of voids at the microlevel — voids of no energetic significance — into the classical continuum theory of linear elastic solids. As Passman (1984) observed, the LEMV may serve as a model of bodies with no initial porosity but may eventually develop pores and expand to the point where the pores take up the whole body remaining elastic throughout. This could be close to the realistic situation in adhesive bonds which may develop pores either during fabrication or during service of the structural element. Of course, we are not concerned with the limit case of the whole adhesive coming unstuck but only in some intermediate stage in which pores are present in the region of adhesion. Reiterating that one of the factors that weakens the bond in composites is the presence of small distributed voids in the interface zone where the adhesive bonds the adherends, we try to look at how these voids change the internal stress field in plates relative to those in perfectly bonded bilaminates. For, observing stress developed can be more conclusive in assessing the presence of weaker or softer inclusions in the bond region of a bilaminate (Kundu and Maslov, 1997).

The theory is briefly presented in the next section and it is applied to glass/glue/glass (G/g/G) symmetric sandwich with assumed porosity in the region of the glue. In the section that follows the results are graphically illustrated and discussed. The paper ends with a few concluding remarks.

2. Theory

We consider a symmetric sandwich plate made of two laminas of material 1, held together by an adhesive layer of material 2. The adhesive layer of thickness $2t^c$ is called the core and the laminas each of thickness t^f held together by the adhesive are called facings. The material of the facings is a linear homogeneous isotropic solid of density ρ^f. Its Lame constants are denoted by λ^f, μ^f. The bond between the two facings is assumed to be imperfect. The imperfection is due to a distribution of vacuous pores or voids throughout the region of adhesion. We represent this thin region by a homogeneous LEMV.

We refer the resulting plate to a Cartesian co-ordinate system with the Ox_1x_2 plate coinciding with midplane of core. The Ox_3 axis is along the plate thickness direction (Fig. 1). The equations of motion in the facings are the well-known Navier equations for linear homogenous isotropic elastic media

$$\mu^f\nabla^2 U^f + (\lambda^f + \mu^f)\nabla\nabla\cdot U^f = \rho^f \ddot{U}^f \quad \text{in } t^c < |x_3| < t^c + t^f \quad \text{with } \mu^f \geqslant 0, \quad 3\lambda^f + 2\mu^f \geqslant 0 \tag{1}$$

where U^f is displacement vector in the facings.

The bond region $|x_3| < t^c$ is filled by the adhesive which includes pores. We model this adhesive zone by a LEMV.

In an LEMV the bulk density (ρ^c) is factored into the product of matrix density (γ^c) and matrix volume fraction (v^c)

$$\text{The bulk density } \rho^c = \frac{\text{adhesive bulk mass}}{\text{adhesive bulk volume}}$$

$$\text{The bulk density } \rho^c = \frac{\text{matrix mass} + \text{mass of voids in adhesive}}{\text{matrix volume}} \times \frac{\text{matrix volume}}{\text{bulk volume}}$$

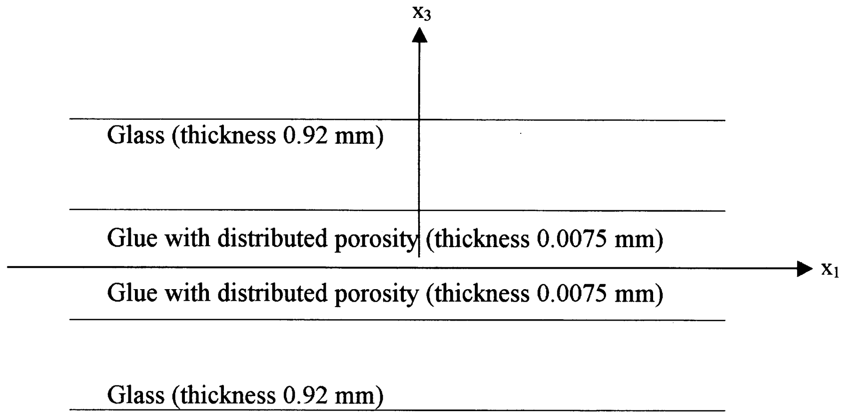

Fig. 1. G/g/G plate and co-ordinate frame.

The bulk density $\rho^c = (\text{matrix density} + 0) \times \text{matrix volume fraction}$

Thus $\rho^c = \gamma^c \nu^c$.

We see that $0 \leqslant \nu^c \leqslant 1$. The case $\nu^c = 1$ corresponds to a material with no voids whereas the case $\nu^c = 0$ corresponds to a material with no strength in extension or shear.

A new variable $\phi(x, t)$ is now defined as

$$\phi^c(x,t) = \nu^c(x,t) - \nu^c_R \tag{2}$$

The independent kinematic variables in the theory of LEMV are the displacement vector $U^c(x, t)$ and the scalar $\phi^c(x, t)$ which describes the change in volume fraction (ν^c) from the reference volume fraction (ν^c_R). The scalar field ϕ endows the LEMV with certain properties akin to those known as standard viscoelastic solids. We know that the adhesives employed in the fabrication of composite laminates are viscoelastic.

The governing equations in core, coupling $U^c(x, t)$ and $\phi^c(x, t)$ are

$$\begin{aligned} &\mu^c \nabla^2 U^c + (\lambda^c + \mu^c)\nabla\nabla \cdot U^c + \beta \nabla \phi = \rho^c \ddot{U}^c \\ &\alpha \nabla^2 \phi^c - \omega \dot{\phi}^c - \xi \phi^c - \beta \nabla \cdot U^c = \rho \kappa \ddot{\phi}^c \quad \text{in } |x_3| < t^c \end{aligned} \tag{3}$$

where α, β, ω, ξ and κ are hypothetical material constants of LEMV in addition to the usual Lame constants λ^c, μ^c. The following thermodynamical relations on these constants exist.

$$\mu^c \geqslant 0, \quad \alpha \geqslant 0, \quad \xi \geqslant 0, \quad \omega \geqslant 0$$

$$(3\lambda^c + 2\mu^c) \geqslant 0, \quad (3\lambda^c + 2\mu^c)\xi \geqslant 3\beta^2$$

The displacements and stresses in the facings have well-known mathematical expressions (Viktorov, 1967). Expressions for the displacement, volume fraction field i.e., the solutions of Eq. (3) and the consequent stresses are obtained using Helmholtz decomposition (Chandrasekhariah, 1987) of a vector field. For the sake of self-containedness we detail all these expressions in both the facings and the core.

$$U^f_1 = \left\{ i\gamma[A^1 \cos(\alpha_1 x_3) + A^2 S(\alpha_1 x_3)] + \beta_1[-A^3 S(\beta_1 x_3) + A^4 C(\beta_1 x_3)] \right\} e^{i(\gamma x_1 - \theta t)} \tag{4}$$

$$U^f_3 = \left\{ \alpha_1[-A^1 S(\alpha_1 x_3) + A^2 C(\alpha_1 x_3)] - i\gamma[A^3 C(\beta_1 x_3) + A^4 S(\beta_1 x_3)] \right\} e^{i(\gamma x_1 - \theta t)} \tag{5}$$

$$U^c_{1e} = \left\{ i\gamma[A^1_e \mathrm{Ch}(m_1 x_3) + A^2_e \mathrm{Ch}(m_2 x_3)] + A^3_e m_0 \mathrm{Ch}(m_0 x_3) \right\} e^{i(\gamma x_1 - \theta t)} \tag{6}$$

$$U^{c}_{3e} = \left\{A^{1}_{e} m_1 \mathrm{Sh}(m_1 x_3) + A^{2}_{e} m_2 \mathrm{Sh}(m_2 x_3) - \mathrm{i}\gamma A^{3}_{e} \mathrm{Sh}(m_0 x_3)\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{7}$$

$$U^{c}_{1b} = \left\{\mathrm{i}\gamma [A^{1}_{b} \mathrm{Sh}(m_1 x_3) + A^{2}_{b} \mathrm{Sh}(m_2 x_3)] + A^{3}_{b} m_0 \mathrm{Sh}(m_0 x_3)\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{8}$$

$$U^{c}_{3b} = \left\{A^{1}_{b} m_1 \mathrm{Ch}(m_1 x_3) + A^{2}_{b} m_2 \mathrm{Ch}(m_2 x_3) + iYA^{3}_{b} \mathrm{Ch}(m_0 x_3)\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{9}$$

$$\tau^{f}_{31} = \left\{\mu^{f}(2\mathrm{i}\gamma\alpha_1[-A^1 S(\alpha_1 x_3) + A^2 C(\alpha_1 x_3)]) + (\gamma^2 - \beta_1^2)\left[A^3 C(\beta_1 x_3) + A^4 S(\beta_1 x_3)\right]\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{10}$$

$$\begin{aligned} \tau^{f}_{33} = \{ & (\lambda^2 + 2\mu^{f})\{\alpha_1^2[-A^1 C(\alpha_1 x_3) - A^2 S(\alpha_1 x_3)] + \mathrm{i}\gamma\beta_1[-A^3 S(\beta_1 x_3) + A^4 C(\beta_1 x_3)]\} \\ & + \lambda^{f}\{-\gamma^2[A^1 C(\alpha_1 x_3) + A^2 S(\alpha_1 x_3)] + \mathrm{i}\gamma\beta_1[-A^3 S(\beta_1 x_3) + A^4 C(\beta_1 x_3)]\}\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \end{aligned} \tag{11}$$

$$\tau^{c}_{31e} = \left\{2\rho^{c} V_2^{c^2} \mathrm{i}\gamma [A^{1}_{e} m_1 \mathrm{Sh}(m_1 x_3) + A^{2}_{e} m_2 \mathrm{Sh}(m_2 x_3)] + \rho^{c} V_2^{c^2} (\gamma^2 + m_0^2) A^{3}_{e} \mathrm{Sh}(m_0 x_3)\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{12}$$

$$\tau^{c}_{33e} = \left\{\rho^{c} V_2^{c^2} \gamma_0 [A^{1}_{e} \mathrm{Ch}(m_1 x_3) + A^{2}_{e} \mathrm{Ch}(m_2 x_3)] - 2\rho^{c} V_2^{c^2} \mathrm{i}\gamma A^{3}_{e} m_0 \mathrm{Ch}(m_0 x_3)\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{13}$$

$$\phi^{c}_{e} = \left(\frac{\rho^{c} V_1^{c^2}}{\beta}\right)\left[r_1 A^{1}_{e} \mathrm{Ch}(m_1 x_3) + r_2 A^{2}_{e} \mathrm{Ch}(m_2 x_3)\right] \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{14}$$

$$\phi^{c}_{b} = \left\{-\left(\frac{\rho^{c} V_1^{c^2}}{\beta}\right)\left[r_1 A^{1}_{b} \mathrm{Sh}(m_1 x_3) + r_2 A^{2}_{b} \mathrm{Sh}(m_2 x_3)\right]\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{15}$$

$$\tau^{c}_{31b} = \left\{2\rho^{c} V_2^{c^2} \mathrm{i}\gamma [A^{1}_{b} m_1 \mathrm{Ch}(m_1 x_3) + A^{2}_{b} m_2 \mathrm{Ch}(m_2 x_3)] + \rho^{c} V_2^{c^2} (\gamma^2 + m_0^2) A^{3}_{b} \mathrm{Ch}(m_0 x_3)\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{16}$$

$$\tau^{c}_{33b} = \left\{\rho^{c} V_2^{c^2} \gamma_0 [A^{1}_{b} \mathrm{Sh}(m_1 x_3) + A^{2}_{b} \mathrm{Sh}(m_2 x_3)] - 2\rho^{c} V_2^{c^2} \mathrm{i}\gamma A^{3}_{b} m_0 \mathrm{Sh}(m_0 x_3)\right\} \mathrm{e}^{\mathrm{i}(\gamma x_1 - \theta t)} \tag{17}$$

The suffixes e and b stand for extensional and bending modes respectively, while f and c are used to denote field variables in the facings and the core respectively. C, S, Ch, Sh denote the circular and hyperbolic functions.

There are seven arbitrary constants in the solution appearing in two sets A^1, A^2, A^3, A^4, A^1_e, A^2_e, A^3_e in the case of extensional waves and A^1, A^2, A^3, A^4, A^1_b, A^2_b, A^3_b in the case of bending waves. γ is wave number and θ is the frequency. The Lamb wave velocity is given by $V = \theta/\gamma$. The different terms used in Eqs. (4)–(17) are defined as

$$\alpha_1^2 = \frac{\theta^2}{V_1^{f^2}} - \gamma^2; \quad \beta_1^2 = \frac{\theta^2}{V_2^{f^2}} - \gamma^2; \quad V_1^{f^2} = \frac{\lambda^{f} + 2\mu^{f}}{\rho^{f}}; \quad V_2^{f^2} = \frac{\mu^{f}}{\rho^{f}}, \; X_3 = x_3 - t^{c}$$

$$m_0^2 = \gamma^2 - \theta^2/V_2^{c^2}, \quad r_{1,2} = m_{1,2}^2 - \gamma^2 + \theta^2/V_2^{c^2}$$

where m_1^2, m_2^2 are the roots of the equation

$$(\gamma^2 - m^2)^2 - [\theta^2/V_2^{c^2} - (1/\alpha^*)(1 - \mathrm{i}\omega^*\theta - k^*\theta^2) + \beta^*](\gamma^2 - m^2) - (\theta^2/V_2^{c^2}\alpha^*)(1 - \mathrm{i}\omega^*\theta - k^*\theta^2) = 0 \tag{18}$$

where

$$V_1^{c^2} = \frac{\lambda^c + 2\mu^c}{\rho^c}, \quad V_2^{c^2} = \frac{\mu^c}{\rho^c}, \quad \frac{\alpha}{\xi} = l_2^2, \quad \omega^* = \omega/\xi, \quad k^* = \frac{\rho^c \kappa}{\xi}, \quad \beta^* = \frac{\beta^c}{\rho^c \alpha V_1^c}, \quad V_3^{c^2} = \alpha^*/k^*,$$
$$V_4^{c^2} = 4\alpha^2/l_0^2\omega^2$$

Further the length parameters l_0, l_1 and l_2 associated with linear elastic materials with voids are defined as

$$l_0 = l_1 l_2 \Big/ \sqrt{l_1^2 - l_2^2 H} \tag{19a}$$

where

$$H = \beta/(\lambda^c + 2\mu^c), \quad l_1 = \sqrt{\alpha/\beta}, \quad l_2 = \sqrt{\alpha/\xi} \tag{19b}$$

Further the coupling constant N is defined as

$$N = \frac{\beta^2}{\xi(\lambda^c + 2\mu^c)} = \frac{l_2^2 H}{l_1^2}$$

Vanishing of N decouples the two kinematical variables namely the displacement and the void volume fraction.

The displacements (4)–(9), stresses (10)–(13) and (16)–(17) and void volume fraction ϕ^c (14)–(15) are subject to the following boundary and interface conditions pertaining to free vibrations of sandwich plate.

$$u_i^f = u_j^c \quad \text{on } |x_3| = t^c \;\; i,j = 1,3 \tag{20a}$$

$$\tau_{3j}^f = \tau_{3j}^c \quad \text{on } |x_3| = t^c \;\; j = 1,3 \tag{20b}$$

$$\phi^c, x_3 = 0 \quad \text{on } |x_3| = t^c \tag{20c}$$

$$\tau_{3j}^c = 0 \quad \text{on } |x_3| = t^c + t^f \;\; j = 1,3 \tag{20d}$$

The Eqs. (4)–(17) are subject to conditions (20). The characteristic equation of the present Lamb wave boundary value problem is obtained as the vanishing of the determinants of the matrices $\mathbf{E}_{ij}$ and $\mathbf{B}_{ij}$ separately for the extensional and bending waves in the plate. The frequency equation for the symmetric and anti-symmetric Lamb modes in G/g/G plate is given by

$$|\mathbf{E}_{ij}| = 0 \tag{21}$$

and

$$|\mathbf{B}_{ij}| = 0 \tag{22}$$

We note down $\mathbf{E}_{ij}$ and $\mathbf{B}_{ij}$ in Appendix A.

3. Numerical results and discussion

The concern of the present paper is to look at the possible utility of the LEMV model developed earlier (Vasudeva and Govinda Rao, 1991, 1992; Vasudeva and Sudheer, in press) in NDT of porosity in the adhesive zone. The model is evaluated in terms of stresses carried by the first few Lamb modes as explained in the introductory section. The frequency equations (21) and (22) for the symmetric and anti-symmetric Lamb modes is for any symmetric bilaminate with classical isotropic facings and LEMV core with arbitrary facing and core thicknesses. We note again that the theory of LEMV is an extension of the classical theory of elasticity to include the mechanical effects of distributed small voids in a solid medium. There are four velocities (V_1^c, V_2^c, V_3^c, V_4^c) in the LEMV medium. V_1^c, V_2^c are the familiar dilatational and distortional wave velocities while the velocities V_3^c, V_4^c are velocities of a wave carrying a change in the volume fraction field at high and low frequencies (Puri and Cowin, 1985). In this paper, for the purpose of comparison, a G/g/G plate is chosen in which the stresses for S_1, A_1, A_2 modes are available in Kundu and Maslov (1997). In the G/g/G plate the mechanical behavior of the facing glass plate is described by the classical theory of elasticity while the pore infested glue region is modeled by an LEMV. The physical–mechanical properties of G/g/G plate are taken in the following manner. The two velocities in the glass are taken from Kundu and Maslov (1997). In the glue region modeled by an LEMV the length parameter (l_2) is fixed at 0.005 mm and the coupling constant (N) is taken to be 0.01 as in Vasudeva and Govinda Rao (1991, 1992) and Vasudeva and Sudheer (in press). The real world glue velocities are taken for V_1^c, V_2^c as in Kundu and Maslov (1997). Since the glue region with its pores is very thin we allowed the glass properties to dominate and let V_3^c, V_4^c assume the wave velocities in glass. Different choices of values for V_3^c, V_4^c along with N and l_2 are possible within the realm of the theory of LEMV (Puri and Cowin, 1985). Such different choices would physically mean qualitative variation in the porosity of the adhesive zone. We confine only for the set of values presented in Table 1.

First, the dispersion of Lamb modes obtained by Kundu and Maslov (1997) is compared with the dispersion of the present model with material constants taken as shown in Table 1. Kundu and Maslov (1997) introduced a scratch in the interface and conducted experiments employing L-scan. The scratch which can be considered as a small void in the interface was not mathematically modeled by Kundu and Maslov i.e., the boundary and interface conditions at the scratch are not theoretically taken care of by them (Kundu, 2000). Experimental results with scratch were however shown. The closeness of the present dispersion spectrum presented in Fig. 2(b) to that given by Kundu and Maslov (1997) as given in Fig. 2(a) is evident. The broader inference of these authors (Kundu and Maslov, 1997) is that voids release stresses and the modes which carry higher stresses can be employed in detecting the presence of softer materials/pores in the interface. We present in Figs. 3–5 the shear and normal stresses for A_1, A_2 and S_1 modes. Along the horizontal we have the normalized depth of plate and along the vertical we have the absolute value of stress. The plate thickness has been normalized as in Kundu and Maslov (1997) to facilitate comparison. Hence 0–1 along the horizontal corresponds to 0–1.855 mm. In these plots the region of adhesion i.e., the core of the sandwich lies between 0.49596 and 0.50404 along the horizontal. In the figures we denote the shear and normal stresses by S_{13} and S_{33} respectively. Fig. 3(a) and (b) shows S_{13} and S_{33} for A_1 mode. Fig. 4(a) and (b) shows S_{13} and S_{33} for A_2 mode and Fig. 5(a) and (b) shows S_{13} and S_{33} for S_1 mode. These are presented at the phase velocity points at which Kundu and Maslov's theory and experiment agree on the dispersion diagram (Fig. 8 of Kundu and Maslov (1997)). For each mode we present three stress versus depth curves for the experimentally tallied phase velocities (pv's) 5.1, 4.58 and 4.16 km/s corresponding respectively to the three different incident angles 17°, 19° and 21°. For anti-symmetric modes the normal stress is zero at the central plane of the glue but at the interface plane it has significant nonzero value. For the same modes the shear stress at the central plane of the glue is more than that at the interface. As we allowed the glass properties to dominate in the adhesive zone, it is as well as a scratch or void is present at the central plane of

Table 1
Material parameters of facings and core

Layer material	Thickness (mm)	Density (g/cm^3)	Velocities (mm/μs)			
			V_1	V_2	V_3	V_4
Glass	0.92	2.25	5.66	3.4	–	–
Adhesive (LEMV)	0.015	1.25	2.8	1.2	5.66	3.4

In core: $N = 0.01$ and $l_2 = 0.005$ mm.

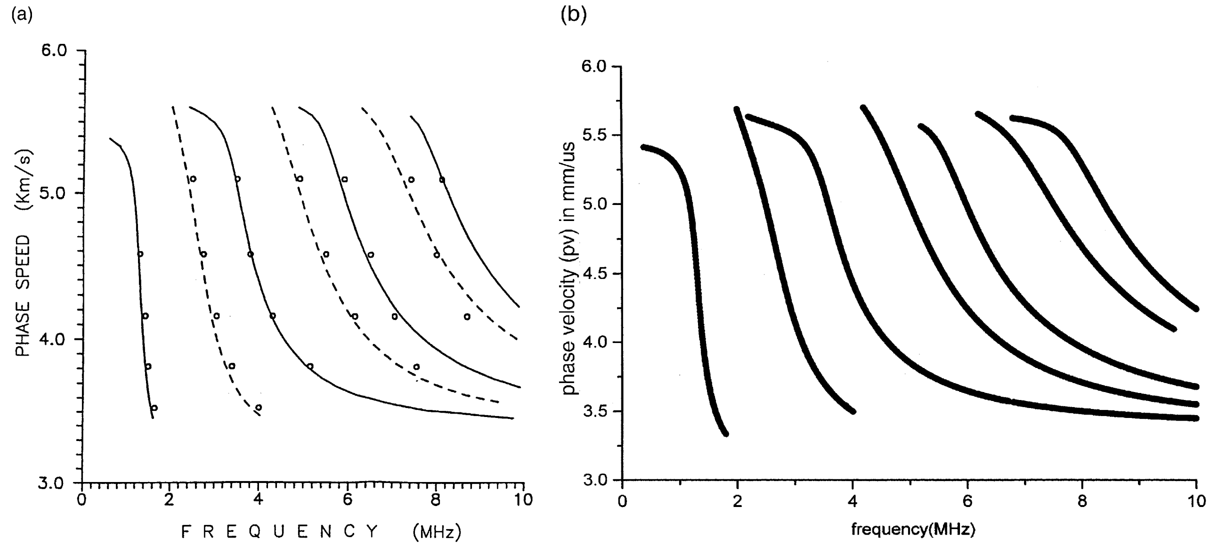

Fig. 2. (a) Dispersion curves in G/g/G plate (after Kundu and Maslov (1997)) circles (○) are experimental data points. (b) Dispersion curves of present model.

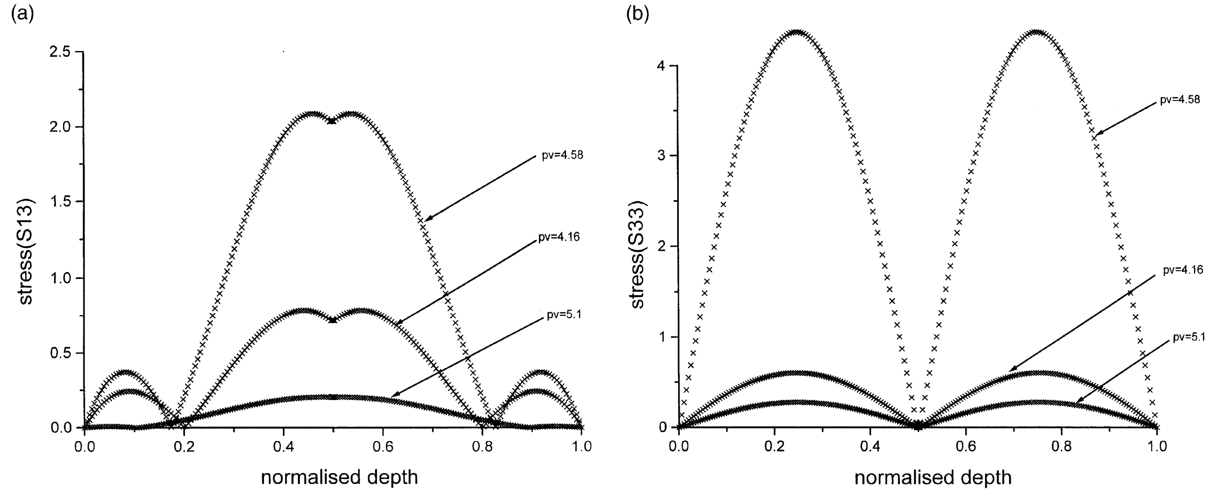

Fig. 3. Variations in (a) S_{13} for A_1 mode and (b) S_{33} for A_1 mode.

glue region. Voids release stresses. Therefore the modes for which the stresses are large—if they can be generated in the lab—can be utilized to detect voids. From the figures it can be reliably said that the A_1 mode can be employed to detect voids as the shear stress is high at the central glue region as in the corresponding case of Kundu and Maslov (1997) with a scratch in the glued zone. A thick more prominent line is seen in the shear stress for S_1 mode in the adhesive region. This is because in the extremely thin region of glue the shear stress is zero at the central plane but rises sharply to a significant nonzero value in the interface. Thus we see that S_1 mode stress records a rise as in Kundu and Maslov (1997). Therefore S_1 mode

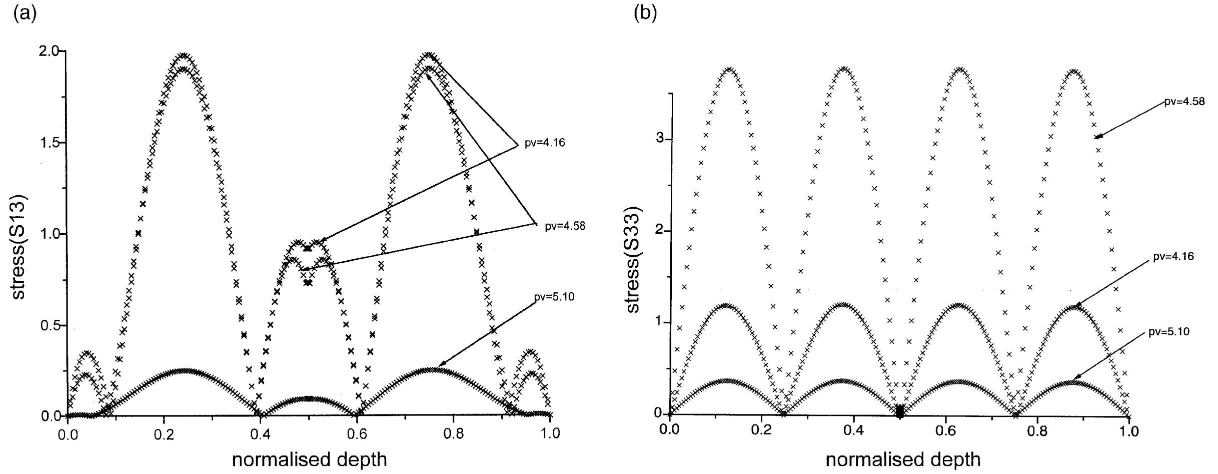

Fig. 4. Variations in (a) S_{13} for A_2 mode and (b) S_{33} for A_2 mode.

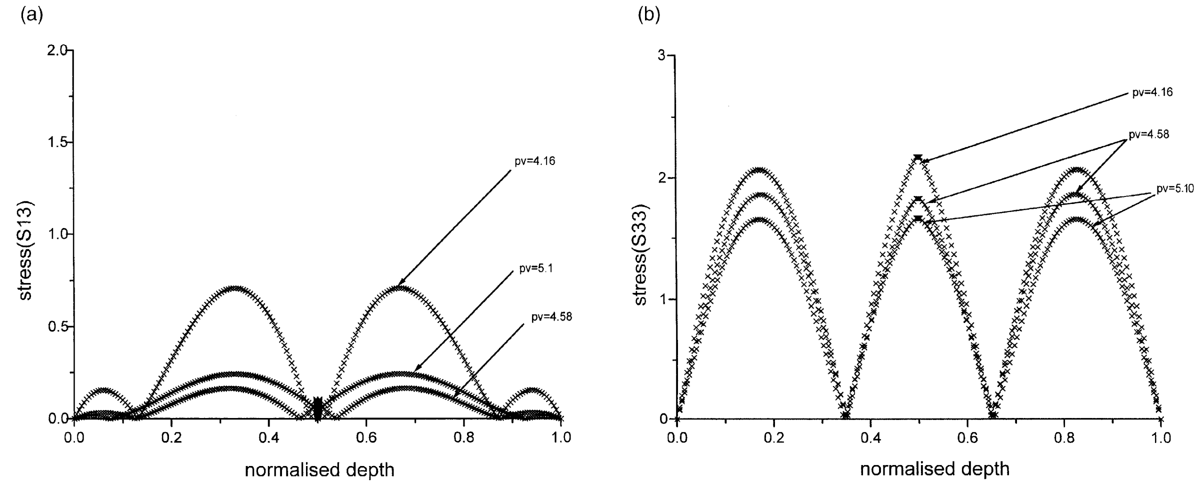

Fig. 5. Variations in (a) S_{13} for S_1 mode and (b) S_{33} for S_1 mode.

too can be utilized to detect porosity. Therefore the LEMV model seems to serve reasonably well in NDT of bondline porosity.

4. Conclusions

Various parameters such as displacements, velocities, energies and stresses carried by Lamb modes in layered plates and cylinders are presently under investigation by researchers interested in employing Lamb modes in NDT of adhesive bonds. The present work is one in such a direction. It attempts to identify the possible use of a theoretical model known as LEMV model in NDT of bilaminates via stress evaluation. The dispersion spectrum and the stresses carried by the first few Lamb modes in a G/g/G plate with assumed porosity in the glue region is presented. The physical–mechanical properties of the plate are borrowed from Kundu and Maslov (1997). The dispersion of the Lamb modes in the frequency range 0–10 MHz with assumed values of the hypothetical parameters of the LEMV are given and it is quite close to the one obtained by Kundu and Maslov (1997). Modeling the adhesive zone — with an assumed presence of voids — by the mathematical theory of LEMV we trace the stresses carried by A_1, S_1 and A_2 modes all along the depth of the bilaminate for various frequency–phase velocity pairs and present them only at those frequency–phase velocity points at which Kundu and Maslov's theory and experiment agree on the dispersion diagram. Though still more work is to be carried out to analyze factors like time of flight and energies carried by the modes, the qualitative results of this limited comparison seem to suggest that A_1 and S_1 modes can be reliably employed to detect voids. The authors believe that this model will interest workers in NDT.

Appendix A

The matrix $\mathbf{E}_{ij}$ is

$$\begin{bmatrix} -p_2S_1 & p_2C_1 & GC_2 & GC_2 & 0 & 0 & 0 \\ GC_1 & GS_1 & q_2S_2 & -q_2C_2 & 0 & 0 & 0 \\ 0 & p_2 & G & 0 & -R_\mathrm{g}m_1\mathrm{Sh}_1 & -R_\mathrm{g}m_2\mathrm{Sh}_2 & R_\mathrm{m}\mathrm{Sh}_0 \\ GS_2 & 0 & 0 & -q_2 & -R_0\mathrm{Ch}_1 & -R_0\mathrm{Ch}_2 & R_\mathrm{g}m_0\mathrm{Ch}_0 \\ \mathrm{i}\gamma & 0 & 0 & \beta_1 & -\mathrm{i}\gamma\mathrm{Ch}_1 & -\mathrm{i}\gamma\mathrm{Ch}_2 & -m_0\mathrm{Ch}_0 \\ 0 & \alpha_1 & -\mathrm{i}\gamma & 0 & -m_1\mathrm{Sh}_1 & -m_2\mathrm{Sh}_2 & \mathrm{i}\gamma\mathrm{Sh}_0 \\ 0 & 0 & 0 & 0 & r_1m_1\mathrm{Sh}_1 & r_2m_2\mathrm{Sh}_2 & 0 \end{bmatrix}$$

The elements of the matrix $\mathbf{B}_{ij}$ which are different from those of the elements of $\mathbf{E}_{ij}$ are

$$B_{35} = -R_\mathrm{g}m_1\mathrm{Ch}_1, \quad B_{36} = -R_\mathrm{g}m_2\mathrm{Ch}_2, \quad B_{37} = -R_\mathrm{m}\mathrm{Ch}_0$$

$$B_{45} = -R_0\mathrm{Sh}_1, \quad B_{46} = -R_0\mathrm{Sh}_2, \quad B_{47} = R_\mathrm{g}m_0\mathrm{Sh}_0$$

$$B_{55} = -\mathrm{i}\gamma\mathrm{Sh}_1, \quad B_{56} = -\mathrm{i}\gamma\mathrm{Sh}_2, \quad B_{57} = -m_0\mathrm{Sh}_0$$

$$B_{65} = -m_1\mathrm{Ch}_1, \quad B_{66} = -m_2\mathrm{Ch}_2, \quad B_{67} = \mathrm{i}\gamma\mathrm{Ch}_0$$

$$B_{75} = r_1m_1\mathrm{Ch}_1, \quad B_{76} = r_2m_2\mathrm{Ch}_2$$

where

$$R_0 = (\mu^\mathrm{c}/\mu^\mathrm{f})\gamma_0, \quad G = (\gamma^2 - \beta_1^2), \quad p_2 = 2\mathrm{i}\gamma\alpha_1, \quad q_2 = 2\mathrm{i}\gamma\beta_1$$

$$\mathrm{Sh}_1 = \mathrm{Sh}(m_1t^\mathrm{c}), \quad \mathrm{Sh}_2 = \mathrm{Sh}(m_2t^\mathrm{c}), \quad \mathrm{Sh}_0 = \mathrm{Sh}(m_0t^\mathrm{c})$$

$$\mathrm{Ch}_1 = \mathrm{Ch}(m_1t^\mathrm{c}), \quad \mathrm{Ch}_2 = \mathrm{Ch}(m_2t^\mathrm{c}), \quad \mathrm{Ch}_0 = \mathrm{Ch}(m_0t^\mathrm{c})$$

$$C_1 = C(\alpha_1t^\mathrm{f}), \quad C_2 = C(\beta_1t^\mathrm{f}), \quad S_1 = S(\alpha_1t^\mathrm{f}), \quad S_2 = S(\beta_1t^\mathrm{f})$$

References

Auld, B.A., 1986. Review on Dieulesaint, E., Royer, D., Elastic waves in solids. IEEE Trans. Sonics and Ultrasonics. 32, 613.
Chandrasekhariah, D.S., 1987. Rayleigh–Lamb waves in an elastic plate with voids. J. Appl. Mech. 54, 509–512.
Cowin, S.C., Nunziato, J.W., 1983. Linear elastic materials with voids. J. Elasticity 13, 125–147.

Dattaguru, B., 1999. Failure analysis of mechanically fastened structural joints, BR Seth Memorial Lecture: 44th Congress of Indian Society of Theoretical and Applied Mechanics.

Dayal, V., Kinra, V.K., 1989. Leaky Lamb waves in an isotropic plate. I—An exact solution and experiments. J. Acoust. Soc. Am. 85, 2268–2276.

Hegemier, D.J., 1989. Adhesive bonded joints, Metals Handbook, ninth ed., vol. 17. ASTM International.

Kundu, T., Maslov, K., 1997. Material interface inspection by Lamb waves. Int. J. Solids Struct. 29, 3885–3901.

Kundu, T., 2000. Personal Communication.

Mal, A.K., Xu, P.C., Bar-Cohen, Y., 1989. Analysis of leaky Lamb waves in bonded plates. Int. J. Eng. Sci. 27, 779–791.

Passman, S.L., 1984. Stress, creep, failure and hysteresis in a linear elastic material with voids. J. Elasticity 14, 201–212.

Paul, H.S., Nelson, V.K., 1996. Flexural vibration of piezoelectric composite hollow cylinder. J. Acoust. Soc. Am. 99 (1), 309–313.

Puri, P., Cowin, S.C., 1985. Plane Waves in linear elastic material with voids. J. Elasticity 15, 167–183.

Vasudeva, R.Y., Rao, P.G., 1991. Influence of voids in interface zones on Lamb wave propagation in composites. J. Acoust. Soc. Am. 89 (2), 516–522.

Vasudeva, R.Y., Rao, P.G., 1992. Influence of voids in interface zones on Lamb wave spectra in fiber reinforced composite laminates. J. Appl. Phys. 71 (2), 612–619.

Vasudeva, R.Y., Sudheer, G. Application of the theory of linear elastic material with voids to NDT of interfaces of bilaminates. J. Adhesion Sci. Tech., in press.

Viktorov, I.A., 1967. Rayleigh and Lamb waves. Plenum Press.

Yang, W., Kundu, T., 1998. Guided waves in multilayered plates for internal defect detection. J. Eng. Mech. 124, 311–318.